$V = Ri$ $\qquad f = by' = bv$

$L\frac{di}{dt} = V$ $\qquad f = ky = k\int v\,dt$

$C\frac{dV}{dt} = i$ $\qquad F = M\frac{d^2y}{dt^2} = M\frac{dv}{dt}$

$F(s) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi ft}\,dt$

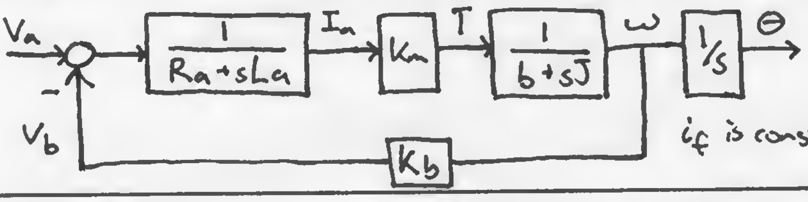
$f(t) = \int_{-\infty}^{\infty} F(s)e^{j2\pi ft}\,dt$

$u(t) \to 1/s$ $\qquad \mathcal{L}(f'(t)) = sF(s) - f(0)$

$e^{-at} \to \frac{1}{s+a}$ $\qquad \mathcal{L}(f''(t)) = s^2F(s) - sf(0) -$

$\delta(t) \to 1$ $\qquad \sin\omega t = \frac{\omega}{s^2+\omega^2}$

ramp $\to \frac{1}{s^2}$ $\qquad \cos\omega t = \frac{s}{s^2+\omega^2}$



$V_b = \underbrace{k_1 k_f i_f}_{K_b : K_m?}\,\omega$

i_f is constant

1st Order System and Step Response:

$\frac{K}{\tau s+1}$, at $t = \tau$, Amplitude is $0.63K$

Routh-Hurwitz: In first column can have no zeros, no sign changes

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\zeta > 1$, overdamped (2 real roots)

$\zeta = 1$, critically damped (2 at same spot)

$\zeta < 1$, underdamped (complex conjugates)

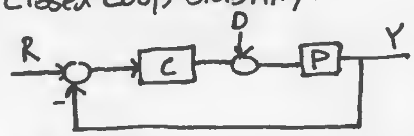
$$p.o. = \exp\left\{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right\} \qquad T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

for $\delta = 2\%$, $T_s = \frac{4}{\zeta\omega_n}$

$\delta = 5\%$, $T_s = \frac{3}{\zeta\omega_n}$

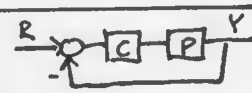
$$\zeta = \frac{\ln(p.o.)}{\sqrt{\pi^2 + \ln^2(p.o.)}} \qquad T_s = \frac{-\ln(\delta) + \frac{1}{2}\ln(1-\zeta^2)}{\zeta\omega_n}$$

Closed Loop Stability:



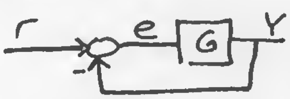
Stable if all $\{R, D\}$ to $\{X, Y\}$ are stable

Sensitivity: $S_P^T = \frac{P}{T}\left(\frac{\partial T}{\partial P}\right)$



Steady State Error: $K_p = \lim_{s\to 0} G(s)$

$K_v = \lim_{s\to 0} sG(s)$



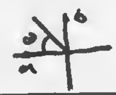
type	Unit Step	Ramp
0	$1/(1+K_p)$	∞
1	0	$1/K_v$
≥ 2	0	0

$\}\ e_{ss}$

$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Theta = a\cos(\zeta)$

$a = \zeta\omega_n$



$$\frac{s+1}{(A) - s(s^2+s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+s+1}$$

Multiply through by (A), solve for $A, B, C = 1, -1, 0$

$$\therefore = \frac{1}{s} - \frac{s}{s^2+s+1}$$

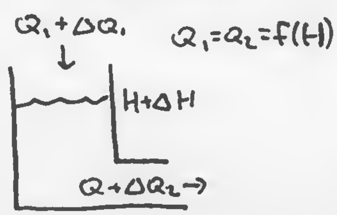
$$\frac{s}{s^2+s+1} = \frac{D}{s+\frac{1}{2}+j\frac{\sqrt{3}}{2}} + \frac{E}{s+\frac{1}{2}-j\frac{\sqrt{3}}{2}}$$

solve for D and E

$\Delta Q_1 - \Delta Q_2 = A\frac{d\Delta H}{dt}$

$\Delta Q_2 = \left.\frac{dQ_2}{dH}\right|_{H=H_0} \Delta H$

$Q_1 = Q_2 = f(H)$



Linear Approximation:

ex: $Q = k(P_1 - P_2)^{1/2}$

$\Delta Q = \frac{dQ}{dP_1} + \frac{dQ}{dP_2}$ $\qquad$ Note: $\frac{dP_1}{dP_1} = \Delta P_1$

$$\therefore \Delta Q = \frac{k}{2(P_1-P_2)^{1/2}}(\Delta P_1 - \Delta P_2)$$

$s = \frac{\ln\delta}{T_s} \pm \frac{\pi}{T_p}$ $\qquad b = \omega_n\sqrt{1-\zeta^2}$

Mason's Formula:

$$\frac{Y}{R} = \frac{\sum P_k\Delta_k}{\Delta}$$

$\Delta = 1 - \sum$ all loop gains

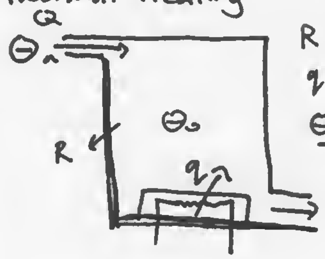
$+ \sum$ all loop gain products of 2 non-touching loops

$- \sum$ all $\cdots$ 3 $\cdots$

$+ \ldots$

$\Delta_k = \Delta$ when kth path is eliminated

Thermal Heating:



S: specific heat

R: Thermal resistance

q: rate of heat flow

$\frac{\Theta_o - \Theta_a}{R}$ = heat loss from walls

Q = fluid flow rate

$Q S \Theta_o - Q S \Theta_a$ = heat going out = $QS(\Theta_o - \Theta_a)$

$\therefore q - QS(\Theta_o - \Theta_a) - \frac{(\Theta_o - \Theta_a)}{R} = C_t \frac{d\Theta(t)}{dt}$

C_t thermal capacitance

$\frac{d\Theta(t)}{dt}$ rate of heat change

solve for q, then $\mathcal{L}$

Note $\Theta(t) \rightarrow \Theta(s)$

$(\Theta_o - \Theta_a) = \Theta(t) \rightarrow \Theta(s)$

$\frac{\Theta(s)}{q(s)} = -$

$\frac{Y}{R} = \frac{CP}{1+CP}$ $\quad \frac{Y}{D} = \frac{P}{1+CP}$ $\quad \frac{X}{D} = \frac{-PC}{1+PC}$